

MEASUREMENT OF VISCOSITIES OF GASES AT HIGH PRESSURE

Part II

Viscosities of Nitrogen and Mixtures of Nitrogen and Hydrogen

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Viscosities of Nitrogen and Mixture of Nitrogen and Hydrogen

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Abstract

Measurements of the viscosities of nitrogen, 80.4% N₂—19.6% H₂ system and 31.9% N₂—68.1% H₂ system have been made within the error of about 1 per cent, using the modified oscillating disc method, at the following conditions:

namely,

pure nitrogen at 25°, 100° and 150°C up to 200 atm,
80.4% N₂—19.6% H₂ at 50° and 100°C up to 200 atm,

and

31.9% N₂—68.1% H₂ at 50°C up to 200 atm and at 100°C up to 100 atm.

From these measurements it was found that $(\partial\eta/\partial\theta)_p$ is positive while $(\partial\eta/\partial p)_\theta$ becomes progressively smaller as the temperature increases and that $(\partial\eta/\partial\theta)_p$ is positive, remains constant over the whole density ranges within the limits of the experimental accuracy and is the function of the temperature alone.

The experimental equations for viscosity-density isotherms were derived using the measured values. These equations reproduce them within the experimental accuracy.

Calculating from the measured values, it was found that the deviations ($\Delta\eta$) of the viscosities of binary mixtures (η_m) from those ($n_N\eta_N + n_H\eta_H$) made equal to the molal average of the viscosities of the components of the mixture are independent of the pressure within the experimental accuracy over the whole measured ranges and are the function of the temperature and the concentration ratio of components.

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Introduction

There have been quite a few studies on the measurement of the viscosities of single gases at high pressure up till the past several years[6], and a theoretical treatment has been proposed by Enskog and others[1] along with some actual measurements[2]. Many methods for this measurement have been suggested[3]. On the other hand, the theoretical treatment of a two component gas under high pressure is extremely difficult, and with the exception of measurements on air there are but a handful of actual measurements[6]. The measurements of the 64.1% H₂—35.9% N₂ system at 30° and 70°C by Boyd[4] are just about qualitative in nature when viewed from the precision of the measurements and other criteria. It may be said that the measured values are worthless to serve as any basis for theorizing

* Numbers in the margin indicate pagination in the original foreign text.

on the viscosities of a gas mixture at high pressure. There are many occasions in industrial areas where gases under high pressures are utilized. When such is the case, there have been attempts to use quasi-critical constants and treat the mixture like a one component gas using one of the methods used in the past for such systems[5]. Any such approach requires some accurately known viscosity values so that the reliability of the method can be tested as well as to establish the limits of precision.

This paper describes work that is a continuation of what was reported in the last paper. It includes the viscosity measurements of a nitrogen-hydrogen system in addition to those on pure nitrogen. Pure nitrogen viscosity measurements were made at 25°, 100°, and 150° up to 200 atm pressure; the 81.4% N₂-19.6% H₂ system measurements were made at 50°C and 100°C up to 200 atm pressure; and measurements of the 31.9% N₂-68.1% H₂ system up to 200 atm at 50°C and up to 100 atm at 100°C.

Measuring Apparatus and Procedure

The apparatus and procedure for these measurements were the same as those detailed in the previous paper in which the method of the oscillating disc suspended between widely spaced plates was followed.

Twenty continuous oscillation amplitudes were measured from which the average was taken for the logarithmic attenuation ratio determination. The total time in seconds for 21 oscillations was divided by the number of oscillations to give the mean period. The watch used was a 30 second watch capable of being read to 0.02 second. The pressure gage used was calibrated from 0-300 atm in one atm intervals. It was an 8" large type Bourdon gage which was standardized with a standard pressure balance. It was capable of readings to 0.1 atm.

The gases used included nitrogen separated from air and electrolytically generated hydrogen. Each gas was passed through NH₄Cl-NH₄OH solution, water, concentrated H₂SO₄, and solid KOH to remove oxygen and then thoroughly dried. The nitrogen purified in this manner was 99.6% pure.

The densities necessary for the viscosity calculations from the measured results were taken from the data of Holbron and Otto[7] and Otto, Michels, and Wouters[8] for nitrogen. The compression ratios measured by Wiebe and Gaddy[9] were used to calculate the densities for the nitrogen-oxygen systems.

Experimental Results

i) Apparatus constant: Measurements were made with several replacements of the oscillating system from which the apparatus constant C and the logarithmic attenuation ratio $2\pi\Delta_0$ due to the internal friction of the suspended line were established. An example of the type of apparatus constant obtained is shown in Table 1.

Here the pressure (p) is in atm., the density (ρ) in g/cc, the periods (T, T_0) in second, and the viscosity (η) in c.g.s. units.

Table 1.

Sample data for the calibration of the apparatus constant.

$$\Delta_0(B) = 0.00008,$$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^6$	C	C_{mean}
C-1	54.5	0.0573 ₁	0.00542 ₂	9.409	9.358		197.4	0.546 ₃	
C-2	43.6	0.0459 ₁	0.00492 ₂	9.398	9.352	9.353	195.4	0.546 ₁	0.547 ₂
C-3	31.0	0.0327 ₁	0.00425 ₂	9.392	9.353		193.4	0.548 ₁	
C-4	20.6	0.0217 ₁	0.00361 ₂	9.384	9.350		191.4	0.548 ₁	

η^* : Values obtained from the interpolation of Michels & Gibson's data.[10]

These measurements were made at 50°C, and the standard viscosity values necessary for the calculations were taken from the data on nitrogen measured by Michels and Gibson[10] just as was done in the preceding paper. The apparatus constants for the other temperatures were calculated from the measured value at 50°C corrected with the coefficient of expansion value of the disc. The oscillating system included (1) disc of 0.925 mm thickness, 28.0 mm diameter (A); (2) same as (A) but with a different type of suspension (B); (3) disc of 0.950 mm thickness, 27.9₄ mm diameter (C); (these are labeled by the respective letters in the tables). The discs were made of V2A stainless steel while the suspending wire was a 0.05 mm diameter, 182 mm long platinum wire.

(ii) The results of the measurements are shown in Tables 2-4. There was no instance when the variance exceeded 0.5% for the series of 3-4 repeated determinations of the logarithmic attenuation ratio made under the same conditions. All the precautions relative to this method[6] were observed, and it was ascertained that these measurements were being made under satisfactory conditions. The errors were below 1%.

The measurements on nitrogen at 25°C were made to test both the reliability and reproducibility of this method, and the data obtained here were compared with the Michels and Gibson[10] data (Table 2). There was very good agreement in the two sets of data, and the validity of the procedure used here was further established.

The experimental values were extrapolated to one atm, and the following results were obtained. N₂ at 100°C $\eta_{1\text{atm}} = 210.9 \cdot 10^{-6}$ (Kleint[11]

$\eta_{1\text{atm}} = 212.5 \cdot 10^{-6}$; Trautz and Baumann[12] $\eta_{1\text{atm}} = 208.5 \cdot 10^{-6}$); N₂ at 150°C $\eta_{1\text{atm}} = 229.0 \cdot 10^{-6}$ (K., $\eta_{1\text{atm}} = 233.0 \cdot 10^{-6}$; T. and B., $\eta_{1\text{atm}} = 228.1 \cdot 10^{-6}$). For the 80.4% N₂-19.6% H₂ system at 50°C, $\eta_{1\text{atm}} = 184.3 \cdot 10^{-6}$; T. and B., $\eta_{1\text{atm}} = 184.1 \cdot 10^{-6}$; 100°C, $\eta_{1\text{atm}} = 206.9 \cdot 10^{-6}$ (K., $\eta_{1\text{atm}} = 208.0 \cdot 10^{-6}$;

T. and B., $\eta_{1\text{atm}} = 205 \cdot 10^{-6}$). For the 31.9% N_2 -68.1% H_2 system, at 50°C, $\eta_{1\text{atm}} = 160.5 \cdot 10^{-6}$ (K., $\eta_{1\text{atm}} = 163.4 \cdot 10^{-6}$; T. and B., $\eta_{1\text{atm}} = 158.0 \cdot 10^{-6}$); 100°C, $\eta_{1\text{atm}} = 176.0 \cdot 10^{-6}$ (K., $\eta_{1\text{atm}} = 178.0 \cdot 10^{-6}$). Thus, there was fairly good agreement with viscosity measurements made at standard atmospheric pressures in the past.

Table 2.
Experimental and calculated data for Nitrogen.
(25°C)

$$\Delta_0(B) = 0.00007,$$

$$C_{25^\circ\text{C}}(B) = 0.547,$$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^6$	$\eta \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$
N-1	86.6	0.0991 ₁	0.00677 ₁	9.395	9.332		194. ₁	195. ₁	195. ₁
N-2	61.5	0.0706 ₁	0.00577 ₁	9.392	9.338		189. ₁	189. ₁	189. ₁
N-3	44.0	0.0506 ₁	0.00495 ₁	9.376	9.330		185. ₁	185. ₁	185. ₁
N-4	20.6	0.0236 ₁	0.00360 ₁	9.369	9.335		181. ₁	181. ₁	181. ₁
N-5	184.5	0.2020	0.01010 ₁	9.428	9.333	9.332	226. ₁	226. ₁	225. ₁
N-6	162.6	0.1804	0.00943 ₁	9.416	9.327		219. ₁	219. ₁	218. ₁
N-7	137.8	0.1548	0.00862 ₁	9.411	9.330		210. ₁	211. ₁	209. ₁
N-8	110.0	0.1250	0.00765 ₁	9.404	9.332		201. ₁	202. ₁	201. ₁
N-9	87.4	0.1000	0.00683 ₁	9.391	9.328		195. ₁	196. ₁	195. ₁

η^* : Values obtained from the interpolation of Michels & Gibson's data.⁽¹⁰⁾

(100°C)

$$\Delta_0(A) = 0.00010,$$

$$C_{100^\circ\text{C}}(A) = 0.542,$$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$
N-10	88.1	0.0784 ₁	0.00666 ₁	9.469	9.407		222. ₁	223. ₁
N-11	61.9	0.0556 ₁	0.00572 ₁	9.454	9.400		218. ₁	218. ₁
N-12	43.5	0.0393 ₁	0.00494 ₁	9.444	9.398		216. ₁	216. ₁
N-13	20.6	0.0187 ₁	0.00368 ₁	9.434	9.400		212. ₁	213. ₁
N-14	186.8	0.1582	0.00949 ₁	9.493	9.404	9.402	243. ₁	243. ₁
N-15	162.5	0.1395	0.00890 ₁	9.483	9.399		239. ₁	237. ₁
N-16	139.7	0.1214	0.00826 ₁	9.481	9.403		233. ₁	232. ₁
N-17	109.6	0.0966 ₁	0.00738 ₁	9.474	9.404		227. ₁	226. ₁

(150°C)

$$\Delta_0(A) = 0.00025,$$

$$C_{150^\circ\text{C}}(A) = 0.541,$$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$
N-18	109.5	0.0844 ₁	0.00740 ₁	9.555	9.487		242. ₁	242. ₁
N-19	86.5	0.0674 ₁	0.00670 ₁	9.534	9.472		238. ₁	238. ₁
N-20	61.3	0.0483 ₁	0.00582 ₁	9.525	9.472		235. ₁	235. ₁
N-21	43.6	0.0346 ₁	0.00510 ₁	9.519	9.473		234. ₁	233. ₁
N-22	20.7	0.0165 ₁	0.00393 ₁	9.510	9.475	9.475	234. ₁	230. ₁
N-23	187.8	0.1391	0.00942 ₁	9.565	9.477		255. ₁	255. ₁
N-24	162.7	0.1221	0.00880 ₁	9.555	9.473		249. ₁	251. ₁
N-25	140.0	0.1064	0.00826 ₁	9.546	9.468		247. ₁	247. ₁

Table 3.

Experimental and calculated data for 80.4%N₂-19.6%H₂ system.

(50°C)

 $\Delta_0(B) = 0.00008$
 $C_{50^\circ C}(B) = 0.547$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^4$	$\eta \cdot 10^4 \text{ Cal.}$
N-H-1	20.6	0.0177 ₄	0.00329 ₄	9.386	9.356	9.356	186. ₀	186. ₃
N-H-2	44.1	0.0377 ₄	0.00446 ₁	9.397	9.356		189. ₄	189. ₃
N-H-3	61.5	0.0523 ₄	0.00513 ₄	9.403	9.355		191. ₇	191. ₃
N-H-4	83.5	0.0705 ₀	0.00588 ₄	9.408	9.354		195. ₂	195. ₄
N-H-5	107.5	0.0898 ₀	0.00661 ₃	9.416	9.354		199. ₄	199. ₀
N-H-6	121.5	0.1008	0.00701 ₄	9.420	9.355		202. ₀	202. ₄
N-H-7	147.0	0.1203	0.00767 ₂	9.434	9.362		207. ₀	208. ₁
N-H-8	162.6	0.1320	0.00805 ₁	9.432	9.357		210. ₄	211. ₄
N-H-9	177.1	0.1425	0.00839 ₇	9.433	9.354		213. ₃	214. ₀
N-H-10	190.4	0.1520	0.00873 ₇	9.439	9.357		218. ₃	218. ₃

(100°C)

 $\Delta_0(B) = 0.00010$
 $C_{100^\circ C}(B) = 0.546$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^4$	$\eta \cdot 10^4 \text{ Cal.}$
N-H-11	20.6	0.0153 ₂	0.00338 ₀	9.442	9.411	9.411	208. ₀	208. ₄
N-H-12	43.6	0.0321 ₄	0.00448 ₅	9.452	9.411		209. ₃	211. ₀
N-H-13	62.5	0.0457 ₂	0.00520 ₄	9.456	9.407		212. ₀	213. ₃
N-H-14	109.5	0.0783 ₃	0.00662 ₄	9.464	9.402		219. ₂	219. ₇
N-H-15	122.1	0.0868 ₀	0.00694 ₂	9.473	9.408		220. ₄	221. ₇
N-H-16	139.3	0.0981 ₃	0.00738 ₃	9.487	9.417		225. ₀	224. ₄
N-H-17	162.4	0.1130	0.00792 ₀	9.490	9.415		229. ₀	228. ₃
N-H-18	196.4	0.1341	0.00869 ₅	9.498	9.416		236. ₄	234. ₀

Table 4.

Experimental and calculated data for 31.9%N₂-68.1 % H₂ system.

(50°C)

 $\Delta_0(C) = 0.00009$
 $C_{50^\circ C}(C) = 0.553$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^4$	$\eta \cdot 10^4 \text{ Cal.}$
N-H-19	20.3	0.00781 ₀	0.00225 ₄	9.329	9.309	9.312	161. ₇	161. ₃
N-H-20	44.5	0.0169 ₄	0.00295 ₂	9.334	9.307		162. ₀	163. ₄
N-H-21	70.1	0.0263 ₃	0.00350 ₆	9.348	9.316		164. ₁	165. ₄
N-H-22	88.1	0.0327 ₀	0.00386 ₀	9.348	9.313		166. ₀	166. ₀
N-H-23	107.4	0.0395 ₂	0.00420 ₂	9.351	9.313		169. ₄	168. ₃
N-H-24	120.8	0.0441 ₂	0.00441 ₃	9.354	9.314		170. ₃	169. ₇
N-H-25	138.6	0.0501 ₁	0.00467 ₄	9.357	9.314		172. ₁	171. ₄
N-H-26	162.7	0.0579 ₀	0.00500 ₁	9.356	9.309		173. ₇	173. ₇
N-H-27	176.5	0.0623 ₀	0.00518 ₃	9.358	9.311		175. ₄	175. ₁
N-H-28	189.4	0.0664 ₃	0.00533 ₀	9.360	9.311		175. ₀	176. ₄

(100°C)

 $\Delta_0(C) = 0.00015$
 $C_{100^\circ C}(C) = 0.552$

Exp. No.	Press.	ρ	Δ	T	T_0	T_{mean}	$\eta \cdot 10^4$	$\eta \cdot 10^4 \text{ Cal.}$
N-H-29	44.5	0.0146 ₅	0.00302 ₀	9.390	9.365	9.357	178. ₀	178. ₃
N-H-30	21.6	0.00719 ₇	0.00240 ₁	9.376	9.354		176. ₀	177. ₃
N-H-31	87.4	0.0281 ₅	0.00388 ₂	9.392	9.357		180. ₃	181. ₀
N-H-32	62.5	0.0204 ₀	0.00344 ₁	9.383	9.352		180. ₄	179. ₄

Examination of the Measured Results

(i) Relation between viscosity and pressure: The isotherms for the relations between the measured viscosity values and pressure are shown in Figs. 1 and 2.

Both Figs. 1 and 2 show that the change in viscosity with pressure $(\partial \eta / \partial p)_\theta$ was larger the lower the pressure just as it is with air[6]. Thus, $(\partial \eta / \partial \theta)_p > 0$ within the limits of this study. It was also observed that $(\partial \eta / \partial p)_\theta$ decreased with increasing mol fraction of hydrogen.

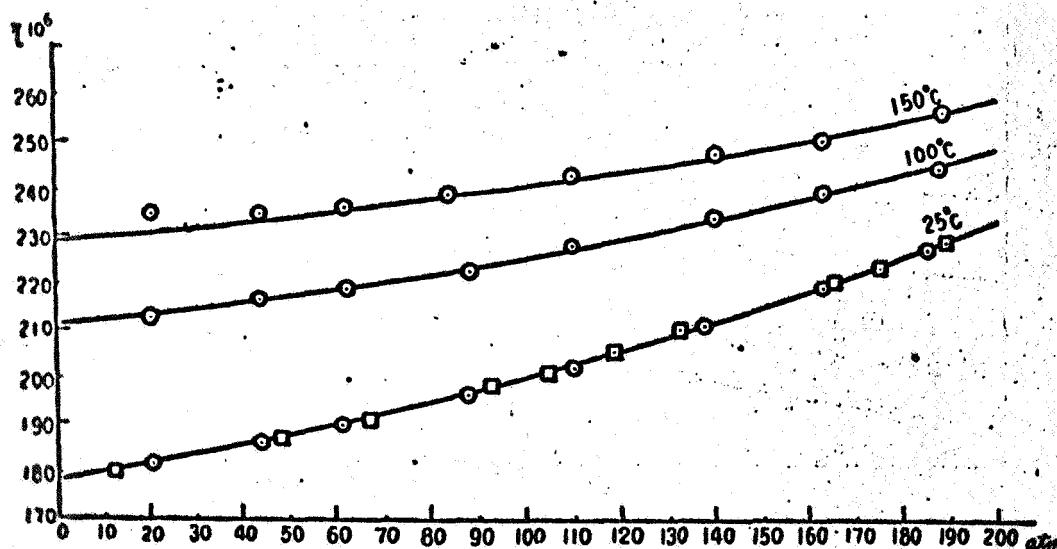


Fig. 1. Relation between Pressure and Viscosity of Nitrogen.

□: Values obtained by Michels & Gibson.(10)

(ii) Relation between viscosity and density: When the isotherms for the measured viscosity versus density relations for the various gases that were obtained in this study are plotted, they describe equally spaced curves within the range of the measurements that were made. The example for the nitrogen system is shown in Fig. 3.

Since the viscosity approaches the viscosity at standard pressure η_0 as the density becomes sufficiently small, it should be possible generally to develop it as a function of the density ρ . In other words

$$\eta = \eta_0 + \alpha \rho + \beta \rho^2 + \dots$$

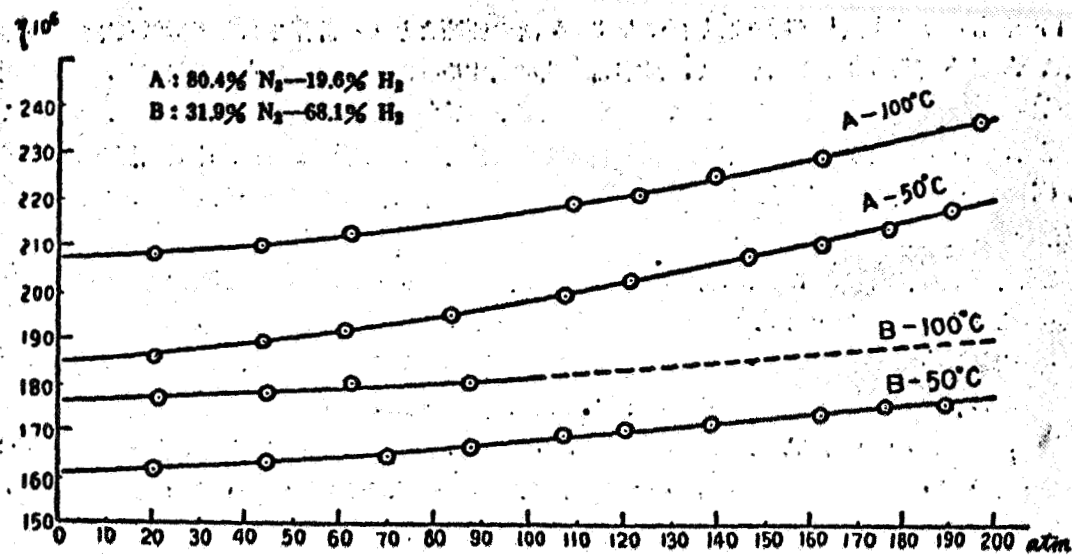


Fig. 2. Relation between Pressure and Viscosity of Binary Mixtures.

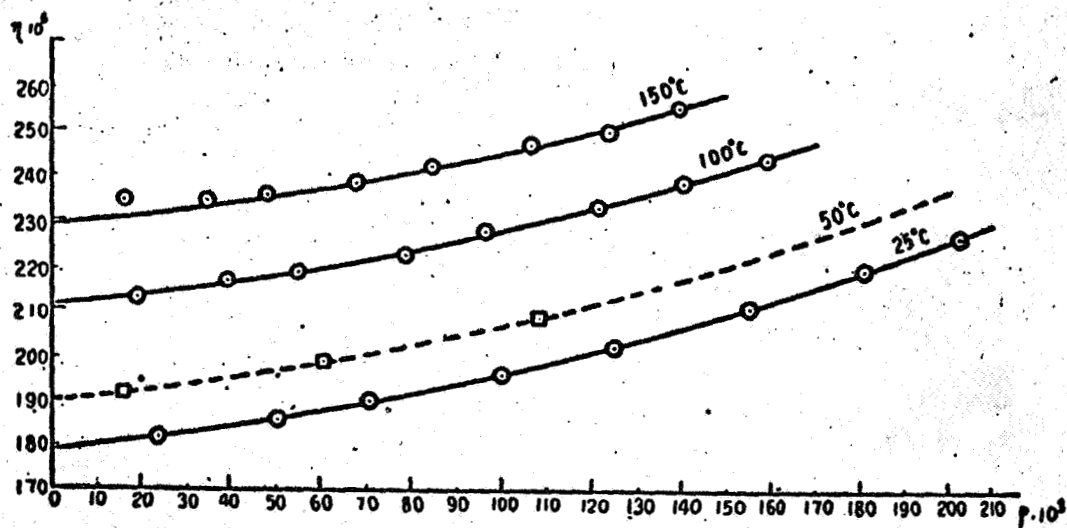


Fig. 3. Relation between Density and Viscosity of Nitrogen.

 \square : Values obtained by Michels & Gibson.

We now take into consideration the equal spacing of the isotherms that was observed in the experimental results and apply the experimental results into the above equation to obtain α , β , and η_0 . We get for the N_2 system

$$\eta_{p,t} = \eta_{a,t} + 0.000103\rho + 0.000643\rho^2 \quad (1)$$

For the 80.4% N_2 -19.6% H_2 system

$$\eta_{p,t} = \eta_{a,t} + 0.000101\rho + 0.000800\rho^2 \quad (2)$$

For the 31.9% N_2 -68.1% H_2 system

$$\eta_{p,t} = \eta_{a,t} + 0.000152\rho + 0.00130\rho^2 \quad (3)$$

The η_{1atm} values that were given before are given as the η_0 values obtained by the above method.

The viscosity values ($\eta_{cal.}$) calculated from the above relations in which the aforementioned values were substituted for $\eta_{a,t}$ are given in the final columns of Tables 2-4. The η_0 for nitrogen at 25°C was taken to be $178.5 \cdot 10^{-6}$ instead of the extrapolated value of $177.5 \cdot 10^{-6}$ from the Michels and Gibson data. The following calculations were made in a similar manner.

It is seen that the above expansions up to ρ^2 give very good agreement with the experimental results. Furthermore, the above relation for nitrogen was found to give very good agreement with the experimental data of Michels and Gibson[10] up to very close to the critical density ($\rho_c = 0.311$ g/cc). This is shown in Table 5.

Table 5.
Comparison of measured and calculated values of viscosity of
nitrogen up to the critical density region.

25°C				50°C				75°C			
p	ρ	$\eta(a) \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$	p	ρ	$\eta(a) \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$	p	ρ	$\eta(a) \cdot 10^6$	$\eta \cdot 10^6 \text{ Cal.}$
14.44	0.01659	180.0	180.4	15.37	0.01623	191.3	191.5	15.37	0.01505	202.2	201.9
48.22	0.05549	186.4	186.3	57.60	0.06049	198.1	198.8	57.61	0.05591	208.8	207.8
104.5	0.1190	200.5	199.9	104.5	0.1083	208.8	208.3	104.5	0.09970	217.5	216.7
212.4	0.2300	236.7	236.3	212.4	0.2067	237.3	238.3	212.4	0.1902	240.8	242.9
320.4	0.3141	276.4	274.3	320.4	0.2875	273.7	272.8	320.3	0.2659	274.6	273.3

(a) Measured values by Michels & Gibson.(10)

The broken line curve in Fig. 3 was obtained from equation (1) using the experimental values of Michels and Gibson extrapolated to 1 atm to be $\eta_0 = 189.6 \cdot 10^{-6}$. The values marked by @ in this figure are the actually measured values of Michels and Gibson. The B-100°C broken line in Fig. 2 shows the portion that was calculated with equation (3).

(iii) Relation between viscosity and composition*: The relation between composition and viscosity at fixed temperature and pressure is given by the following equation.

$$\eta_m = n_N \eta_N + n_H \eta_H + \Delta(n)$$

Here η_m is the viscosity of the gas mixture of n_N mol fraction nitrogen and n_H mol fraction hydrogen while η_N and η_H are the viscosities of pure nitrogen and hydrogen at the given pressure and temperature.

When the viscosity of a mixed gas is expressed in this manner, $\Delta(n)$ is obviously a function of composition, temperature, and pressure.

Next, the experimental results from the preceding section were used to determine the effect of pressure on $\Delta(n)$ at 50°C and 100°C. It was assumed that the viscosity versus density relations for pure hydrogen given before were valid even for this particular area and we used the Gibson data [13] for 25°C up to 300 atm and the Trautz and Kurz[14] and Sutherland and O. Maass[15] data for 50° and 100°C at one atm for the calculations.

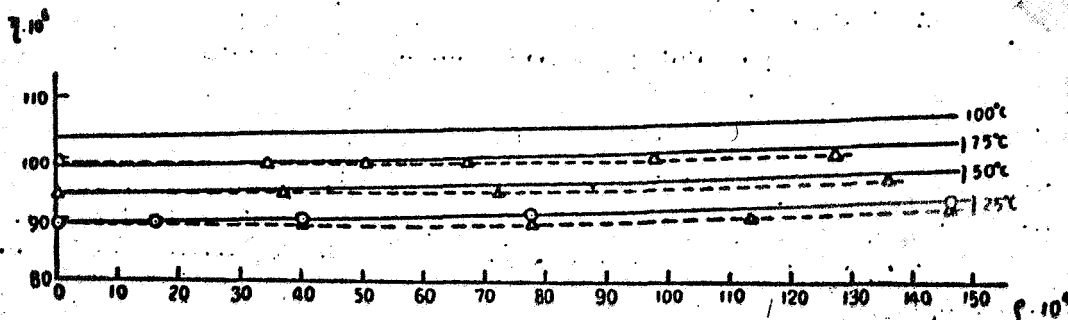


Fig. 4. Relation between Density and Viscosity of Hydrogen.

○ Values obtained by Gibson.

△ Values obtained by Kuss.

The solid lines in Fig. 4 are the isotherms of the viscosity versus density relations obtained in this manner. The data of Kuss[16] for 25°, 50°, and 75°C are given as the broken lines. While the Kuss lines at the different temperatures are equally spaced, there is a 2% precision in these values from which it can be said that the agreement with the calculated results is good.

* The viscosity values used for the examination of this relation were calculated from equations (1)-(3) mentioned before while the values for hydrogen were obtained from the viscosity-density relation that is given later.

The relation between $\Delta_{(n)}$ and pressure calculated from aforementioned experimental results are given in Table 6. It is seen from these results that $\Delta_{(n)}$ is a function of composition and temperature and is independent of the pressure at least within the confines of experimental error.

It then became possible to construct the isobars for the viscosity versus composition for a two component system using the relation between viscosity and composition at 1 atm and the viscosity versus pressure relation taking into account the associated precisions.

The above data were used to construct the isobars for the viscosity versus composition relation at 1 atm using the theory of Hirschfelder et al., [17] and the $\Delta_{(n)}$ values were calculated from the 1 atm isobars*. It has been shown that this theory fits very well with data from many investigators, and the figure shown later shows a fairly good agreement with the data obtained by the author. These $\Delta_{(n)}$ values and the high pressure viscosity values for single gases were used to construct the isobars at 50°C and 100°C taken at 50 atm intervals. These are shown in Figs. 5 and 6. The $\Delta_{(n)}$ -curve is placed under the family of isobars in each figure.

The 1 atm isobar shown in these figures were calculated with the Hirschfelder theory, and the points marked with \square show these calculated points. The \circ points are the author's measured values. It is seen that there is agreement to within 1% between the experimentally measured values and the isobars at least within the limits of this study.

The viscosity of the 31.9% N_2 -68.1% H_2 system at 150 and 200 atm of $186.2 \cdot 10^{-6}$ and $191.0 \cdot 10^{-6}$ respectively taken from Fig. 6 and the values of $185.9 \cdot 10^{-6}$ and $190.0 \cdot 10^{-6}$ from the broken line portion of B-100°C in Fig. 2 which were calculated with equation (3) show good agreement. It is inferred that the broken line portion of B-100°C represents the true values.

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* Following their practice, the force constants (ϵ/k , r_0) between like molecules were obtained from the viscosity and the force constants between unlike molecules from the diffusion coefficient[17].

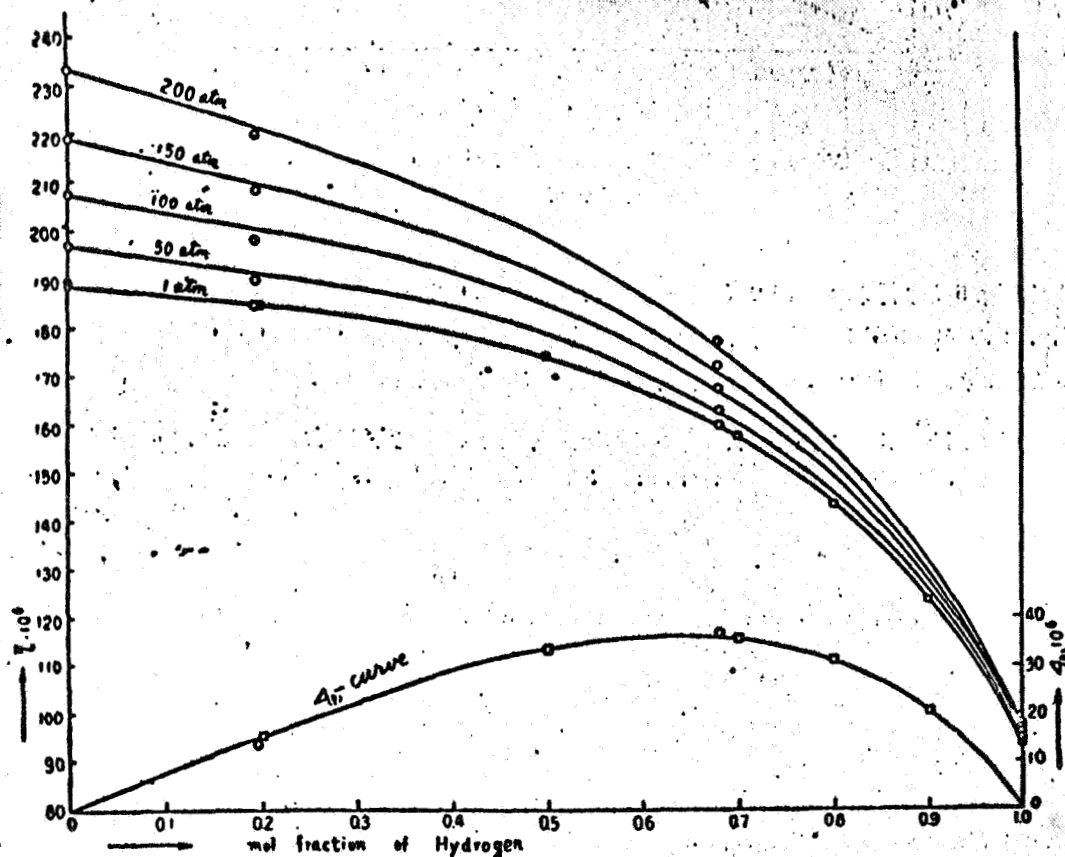


Fig. 5. Viscosity isobar of nitrogen and hydrogen system as a function of mol fraction of hydrogen at 50°C.

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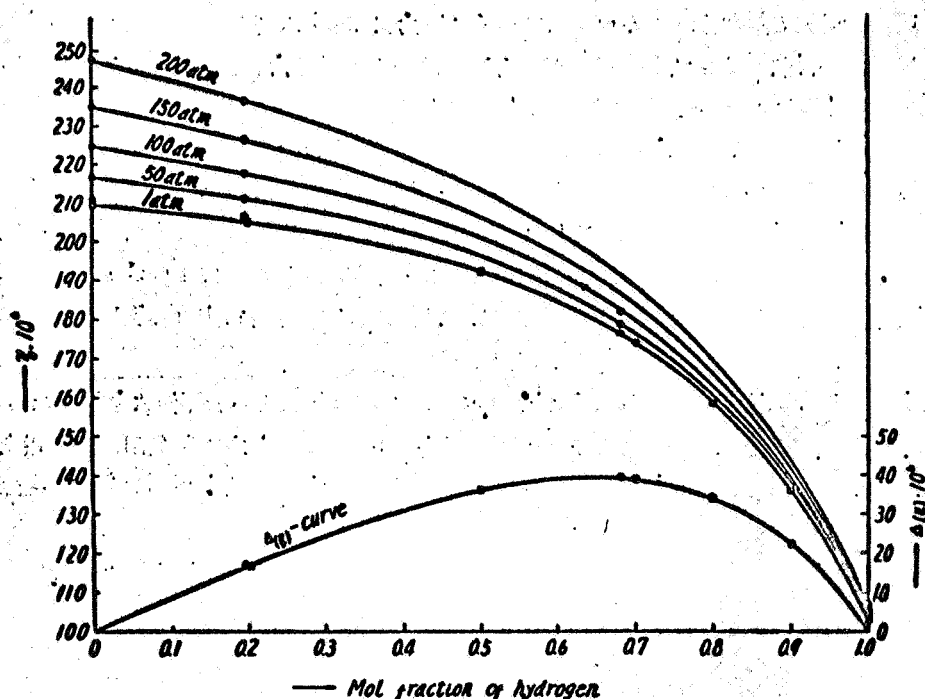


Fig. 6. Viscosity isobar of nitrogen and hydrogen system as a function of mol fraction of hydrogen at 100°C.

Note:

In the past, the $\eta/\eta^* - P_r - T_r$ curve (η^* = some standard viscosity) was reliable only to about $T_r(T/T_c) \approx 3.0$ because of the lack of experimental data. There was no curve which could give reliable values above this T_r .

Using the experimental data noted before and the data on the viscosity of hydrogen reported by Gibson and Kuss, the $\eta/\eta_0 - P_r - T_r$ curve (η_0 : viscosity at standard pressure at the respective temperature) were derived following the Comings and Egly procedure. These were plotted, and the critical temperature limit was expanded from $T_r = 2.80$ to $T_r = 7.25$.

It was found that the nitrogen-hydrogen system could be duplicated within 1% error of the iso- T_r curves for the same η/η_0 versus P_r relations. There are no other data available except the author's work on air at 150°C which come within the scope represented by this figure. The values on this figure come within 1-2% of the measured results obtained by the author.

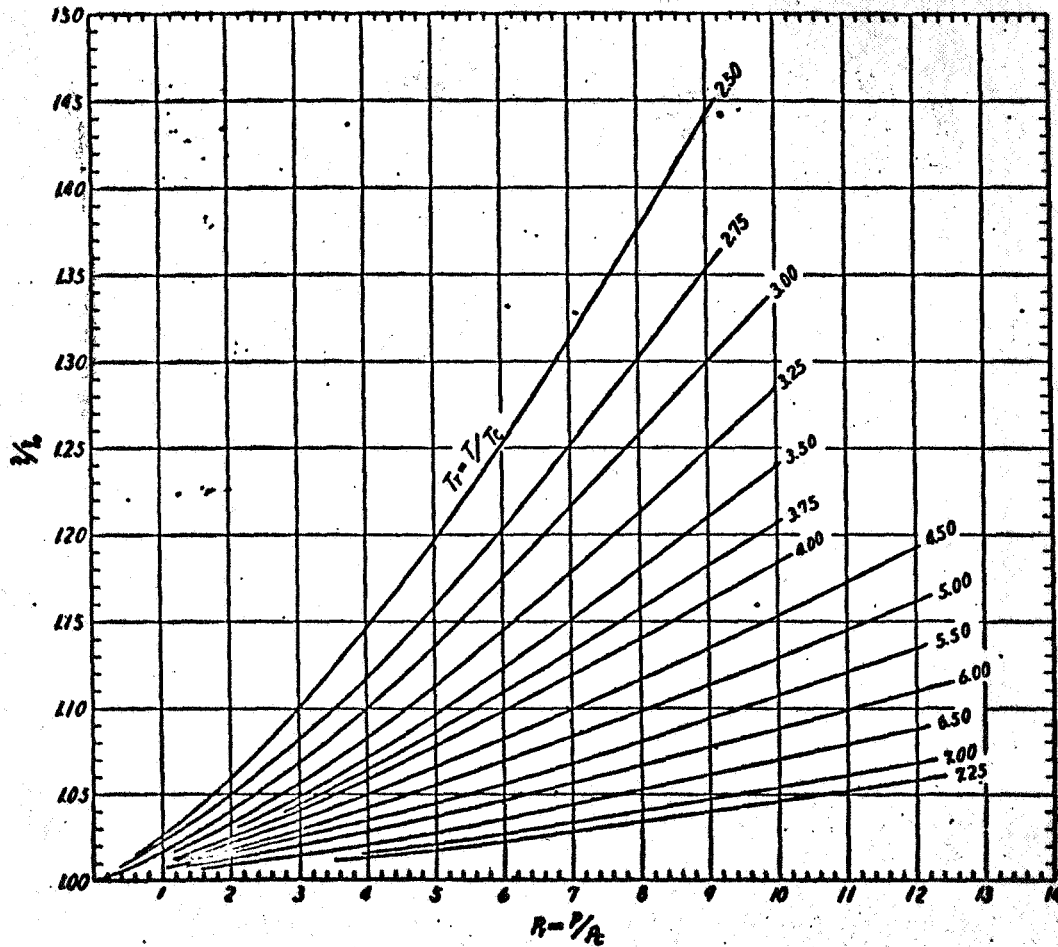


Fig. 7. Generalized Reduced Viscosities.